

Teleportation of entangled states without Bell-state measurement

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In a recent paper [Phys. Rev. A **70**, 025803 (2004)] we presented a scheme to teleport an entanglement of zero- and one-photon states from a bimodal cavity to another one, with 100% success probability. Here, inspired on recent results in the literature, we have modified our previous proposal to teleport the same entangled state without using Bell-state measurements. For comparison, the time spent, the fidelity, and the success probability for this teleportation are considered.

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Quantum teleportation, first suggested by Bennett *et al* [1], is one of the cornerstones of the quantum computation [2] and quantum communication [3, 4]. The crucial ingredient characterizing this phenomenon is the transfer of information between non interacting systems, at the expense of a quantum channel. It has received great attention since its pioneer proposal, mainly after its experimental realizations, e.g.: by Bouwmeester *et al.* [5], Boschi *et al.* [6], Lombardi *et al.* [7], all them using pairs of entangled photons by the process of parametric down-conversion; Furusawa *et al.* [8], using entangled squeezed states; D. Fattal *et al.* [9], using a quantum dot single-photon source. Since then, various schemes have been suggested to implement the teleportation process in different contexts, such as trapped ions [10], running wave fields [11, 12, 13, 14, 15], and trapped wave fields inside high-Q cavities [16, 17, 18].

To implement teleportation between two distinct separated points, usually named as Alice and Bob, one should: i) prepare a state to be teleported; ii) prepare the nonlocal quantum channel; iii) make a Bell measurement (Alice); iv) communicate (to Bob), by classical channel, the measured result. The main experimental challenge consists in the so-called Bell state measurement [1], performed on the Bell operator basis for the particle (or field) whose state is to be teleported, plus its partner composing the quantum channel. The same procedure is employed when we are concerned with teleportation of entangled states with major interest for quantum information processing [17, 19, 20].

Various schemes for teleportion differing from the original protocol - in the sense that Bell-state measurement is not employed - have been proposed: in Ref.[21], Vaidman considered both a spin state and a system with continuous variable, and presented a “cross measurement” method, thus obtaining a two-way teleportation. In Ref. [22], de Almeida *et al.* proposed a scheme to teleport a superposition of coherent states from a high- Q cavity to another with 100% fidelity. The method includes damping effects and creation of the state to be teleported. In

Ref.[23], Zheng refers to the (approximated) teleportation of the superposition of zero- and one-photon states from a high- Q cavity to another, with fidelity near 99%. Both the procedures in Refs.[22, 23] considered a single mode of a trapped field interacting with a single two-level atom via the Jaynes-Cummings Hamiltonian. Soon after Ye and Guo [24] have treated the teleportation of an unknown atomic state in cavity QED; accordingly, the advantage of this scheme is that only virtual field excitations occur with the passage of atoms through the cavity, hence no transfer of quantum information happens between the atoms and the cavity when the cavity is initially assumed in the vacuum state. Also, as in Ref.[22], the state teleportation without Bell state measurements occurs exactly, with 50% of success probability.

Meanwhile [18], a proposal for teleporting an entanglement of zero- and one-photon states was presented with 100% success probability and 100% fidelity. This scheme requires a collection of two kinds of two-level atoms, a three-level atom in a ladder configuration driven by a classical field, Ramsey zones plus selective atomic detectors. As usually, Bell-measurements were employed. Here we will neglect the use of Bell measurements, as done in Refs.[21, 22, 23, 24], to simplify the teleportation scheme of Ref.[18]. Our producere is inspired on the Ref.[23] to implement an approximated conditional teleportation of an unknown entanglement of zero- and one-photon states from a bimodal high-Q cavity to another such one. This scheme employs two two-level (resonant, Rydberg) atoms, Ramsey zones and a selective atomic state detector. The success probability of this scheme coincides with that of the original protocol (25%) if we restrict the measurements on the Bell basis to only one of the four Bell states. Differently from [18], an additional atom is no longer used, which simplifies the teleportation of zero- and one-photon entanglement states. As assumed in Ref. [18, 23], the whole losses due to atomic spontaneous emission and dissipation in the cavities were neglected. Actually, since the decoherence time is of the same order of the lifetimes for the qubits in a high- Q cavity and the (spontaneous) atomic decay, the experimental implementation of the present scheme should be made in a time interval lesser than the 10^{-2} s, a typical time for both decoherence and damping of atomic and

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cavity qubits [25].

Fig.1 displays the setup of experimental proposal: S_A represents “Source of Atoms”, “Excitation” prepares a highly excited (Rydberg) atom, “ C_1 ” (“ C_2 ”) represents the first (second) bimodal cavity and D_e (D_g) stands for atomic ionization detector for the state $|e\rangle$ ($|g\rangle$). The two bimodal cavities sustain two nondegenerate orthogonally polarized modes [26], A and B in C_1 (C and D in C_2).

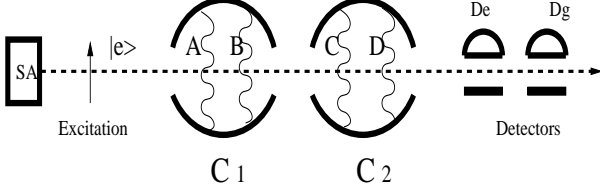


FIG. 1: Scheme of the experimental setup required for teleportation of an entanglement of zero- and one-photon state without using Bell states.

To perform the teleportation of zero- and one-photon entangled states between two high-Q cavities we assume the second cavity C_2 initially prepared in the state [18]

$$|\psi\rangle_{C_2} = \alpha |1\rangle_C |0\rangle_D + \beta |0\rangle_C |1\rangle_D, \quad (1)$$

where α and β are unknown coefficients. First, we send a two-level atom, in an excited state $|e\rangle_1$, through the initially empty cavity C_1 . The atom-1 interacts resonantly with mode A according to the Jaynes-Cummings Hamiltonian [27]. We note that when interaction between the atom and one of the two modes takes place, the other mode remains unaffected, as shown in Ref.[18]. We have, for the *on-resonant* interaction,

$$H_{on} = \hbar g (\sigma^- a^\dagger + \sigma^+ a), \quad (2)$$

where a^\dagger and a stand for the creation and annihilation operator for the cavity modes A, B, C, D ; the σ 's represent the Pauli operators; and g is the atom-field coupling parameter. The state describing the system in C_1 after the atom-field interaction evolves from the initial state $|0\rangle_A |0\rangle_B |e\rangle_1$ to

$$|\varphi(t)\rangle = \cos(gt) |0\rangle_A |0\rangle_B |e\rangle_1 - i \sin(gt) |1\rangle_A |0\rangle_B |g\rangle_1, \quad (3)$$

and setting $gt = \pi/4$, we obtain

$$|\varphi(\pi/4g)\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B |e\rangle_1 - i |1\rangle_A |0\rangle_B |g\rangle_1). \quad (4)$$

Now, the state describing the system, including the cavity C_2 , is

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B |e\rangle_1 - i |1\rangle_A |0\rangle_B |g\rangle_1) \otimes (\alpha |1\rangle_C |0\rangle_D + \beta |0\rangle_C |1\rangle_D). \quad (5)$$

After the evolution due to the resonant interaction of atom-1 with mode C of cavity C_2 governed by the

Hamiltonian (2), the state describing the whole system results in

$$\begin{aligned} |\Phi(t')\rangle = & \frac{\alpha}{\sqrt{2}} \cos(\sqrt{2}gt') |0\rangle_A |0\rangle_B |1\rangle_C |0\rangle_D |e\rangle_1 \\ & - \frac{i\alpha}{\sqrt{2}} \sin(\sqrt{2}gt') |0\rangle_A |0\rangle_B |2\rangle_C |0\rangle_D |g\rangle_1 \\ & - \frac{i\alpha}{\sqrt{2}} \cos(gt') |1\rangle_A |0\rangle_B |1\rangle_C |0\rangle_D |g\rangle_1 \\ & - \frac{\alpha}{\sqrt{2}} \sin(gt') |1\rangle_A |0\rangle_B |0\rangle_C |0\rangle_D |e\rangle_1 \\ & + \frac{\beta}{\sqrt{2}} \cos(gt') |0\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D |e\rangle_1 \\ & - \frac{i\beta}{\sqrt{2}} \sin(gt') |0\rangle_A |0\rangle_B |1\rangle_C |1\rangle_D |g\rangle_1 \\ & - \frac{i\beta}{\sqrt{2}} |1\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D |g\rangle_1. \end{aligned} \quad (6)$$

When detecting the atom-1 in the state $|e\rangle_1$, $|\Phi(t')\rangle$ collapses in the form

$$\begin{aligned} |\Phi(t')\rangle_{col} = & N_1 \left[\alpha \cos(\sqrt{2}gt') |0\rangle_A |0\rangle_B |1\rangle_C |0\rangle_D \right. \\ & - \alpha \sin(gt') |1\rangle_A |0\rangle_B |0\rangle_C |0\rangle_D \\ & \left. + \beta \cos(gt') |0\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D \right]. \end{aligned} \quad (7)$$

where N_1 is a normalization factor.

Next the atom-2, previously prepared in the state $|e\rangle_2$, interacts with the mode B in C_1 , leaving the entangled state of the whole system in the form (with $gt = \pi/4$),

$$\begin{aligned} |\chi(\pi/4g)\rangle = & \alpha [\cos(\sqrt{2}gt') |0\rangle_A |0\rangle_B |1\rangle_C |0\rangle_D |e\rangle_2 \\ & - i \cos(\sqrt{2}gt') |0\rangle_A |1\rangle_B |1\rangle_C |0\rangle_D |g\rangle_2 \\ & - \sin(gt') |1\rangle_A |0\rangle_B |0\rangle_C |0\rangle_D |e\rangle_2 \\ & + i \sin(gt') |1\rangle_A |1\rangle_B |0\rangle_C |0\rangle_D |g\rangle_2] \\ & + \beta [\cos(gt') |0\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D |e\rangle_2 \\ & - i \cos(gt') |0\rangle_A |1\rangle_B |0\rangle_C |1\rangle_D |g\rangle_2]. \end{aligned} \quad (8)$$

Then the atom-2 interacts with the mode D in C_2 which leads the Eq.(8) to

$$\begin{aligned} |v(t')\rangle = & \alpha \left[\cos(\sqrt{2}gt') \cos(gt') |0\rangle_A |0\rangle_B |1\rangle_C |0\rangle_D |e\rangle_2 \right. \\ & - i \cos(\sqrt{2}gt') \sin(gt') |0\rangle_A |0\rangle_B |1\rangle_C |1\rangle_D |g\rangle_2 \\ & - i \cos(\sqrt{2}gt') |0\rangle_A |1\rangle_B |1\rangle_C |0\rangle_D |g\rangle_2 \\ & + i \sin^2(gt') |1\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D |g\rangle_2 \\ & + i \sin(gt') |1\rangle_A |1\rangle_B |0\rangle_C |0\rangle_D |g\rangle_2 \\ & - \sin(gt') \cos(gt') |1\rangle_A |0\rangle_B |0\rangle_C |0\rangle_D |e\rangle_2] \\ & + \beta \left[\cos(gt') \cos(\sqrt{2}gt') |0\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D |e\rangle_2 \right. \\ & - i \cos(gt') \sin(\sqrt{2}gt') |0\rangle_A |0\rangle_B |0\rangle_C |2\rangle_D |g\rangle_2 \\ & - i \cos^2(gt') |0\rangle_A |1\rangle_B |0\rangle_C |1\rangle_D |g\rangle_2 \\ & \left. - \cos(gt') \sin(gt') |0\rangle_A |1\rangle_B |0\rangle_C |0\rangle_D |e\rangle_2 \right]. \end{aligned} \quad (9)$$

Finally, detection of atom-2 in the state $|e\rangle_2$ projects the whole state as follows

$$\begin{aligned} |\psi'\rangle_{col} = & N_2 \left[\alpha \cos(\sqrt{2}gt') \cos(gt') |0\rangle_A |0\rangle_B |1\rangle_C |0\rangle_D \right. \\ & + \beta \cos(gt') \cos(\sqrt{2}gt') |0\rangle_A |0\rangle_B |0\rangle_C |1\rangle_D \\ & - \beta \cos(gt') \sin(gt') |0\rangle_A |1\rangle_B |0\rangle_C |0\rangle_D \\ & \left. - \alpha \sin(gt') \cos(gt') |1\rangle_A |0\rangle_B |0\rangle_C |0\rangle_D \right]. \quad (10) \end{aligned}$$

where N_2 is a normalization factor. Now, according to the protocol in Ref.[23], we adjust $gt' = 7\pi/4$ implying $\cos(\sqrt{2}gt') \simeq 0.078 \simeq 0$, which leads the state in Eq.(10) to the form,

$$|\psi\rangle_{C_1 C_2} = (\alpha |1\rangle_A |0\rangle_B + \beta |0\rangle_A |1\rangle_B) \otimes |0\rangle_C |0\rangle_D \quad (11)$$

in which we recognize the teleportation of the initial state in the cavity C_2 (see Eq.1) to the cavity C_1 , namely,

$$|\psi\rangle_{C_1} = \alpha |1\rangle_A |0\rangle_B + \beta |0\rangle_A |1\rangle_B. \quad (12)$$

This teleportation is attained with fidelity

$$\begin{aligned} F &= |_{C_1 C_2} \langle \psi | \psi' \rangle_{col}|^2 \\ &= \frac{\sin^2(gt')}{\cos^2(\sqrt{2}gt') + \sin^2(gt')} \simeq 0.97. \quad (13) \end{aligned}$$

Note that the fidelity does not depend on the arbitrary field coefficients α and β , as occurs in the Ref.[23]. Here the direct measurements of the two atoms in the excited states, $|e\rangle_1$ and $|e\rangle_2$, substitute the Bell-state measurements projecting a desired state in the cavity C_1 with fidelity 100%.

In summary, we have inspired by the scheme of Zheng [23] to teleport an arbitrary zero- and one-photon entangled states between two bimodal high-Q cavities, using two two-level (Rydberg) atoms. Instead of obtaining exact teleportation, as found in the methods using the original protocol, here teleportation is attained approximately, with 25% success probability (and 97% fidelity), lesser than the 100% (and 100%) found in [18]. However, in [18] the 100% accuracy came from the use of two additional atoms to perform unitary operations by Bob, as required by the original protocol. By limiting the original protocol to only one of the four Bell states (and ignoring the atom preparing the state to be teleported), one reduces to 25% the success probability of the scheme in [18] - while maintaining 100% fidelity. Since the present procedure requires lesser number of atoms and, consequently, of selective atomic detectors, it becomes more economical than that of [18]: it reduces the time spent in the teleportation and decoherence effects caused by unavoidable interactions of the system with its environment, thus increasing its experimental feasibility. In additional, this time to complete the teleportation is of same order of that found in [23], both being lesser than the decoherence time.

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